A SIMPLE THEORETICAL PREDICTION OF THE DATA CORRESPONDING TO OBSERVATIONALLY ESTIMATED VALUE OF COSMOLOGICAL CONSTANT

Vladan Panković[*,♯],Simo Ciganović[♯],
Jovan Ivanović[♯], Rade Glavatović[◇], Petar Grujić[□]
[*]Department of Physics, Faculty of Sciences, 21000 Novi Sad,
Trg Dositeja Obradovića 4. , Serbia, vdpan@neobee.net
[♯]Gimnazija, 22320 Indjija, Trg Slobode 2a, Serbia
[◇] Military-Medical Academy, 11000 Belgrade, Crnotravska 17., Serbia
[□] Institute of Physics, 11 000 Belgrade, P. O. Box 57, Serbia, grujic@phy.bg.ac.yu

PACS number : 98.80.-k, 98.80.Qc

Abstract

In this work a satisfactory, simple theoretical prediction of the data corresponding to observationally (by fine tuning condition) estimated value of the cosmological constant is given. It is supposed (in conceptually analogy with holographic principle) that cosmological constant, like classical surface tension coefficient by a liquid drop, does not correspond to a volume (bulk) vacuum mass (energy) density distribution but that it corresponds to a surface vacuum mass (energy) density distribution. Then form of given surface mass distribution and fine tuning condition imply observed growing (for ~ 61 magnitude order) of the scale factor (from initial, corresponding to Planck length, to recent, at the beginning of the cosmic acceleration, corresponding to 10 Glyr length).

As it is well-known (see for example [1]), Friedmann equation without cosmological constant term

$$(\frac{1}{a}\frac{da}{dt})^2 + \frac{kc^2}{a^2} = G\frac{8\pi}{3}\rho \tag{1}$$

(where a represents the scale factor of the universe, k - curvature constant (that equals 1 for closed, 0 for flat and -1 for open universe), c - speed of light, G - Newtonian gravitational constant, ρ - total mass density) can be formally interpreted by classical, Newtonian mechanics and gravitation in the following way. Namely, equation (1) can be equivalently, simply transformed in the following equation

$$\frac{1}{2}(m\frac{dr}{dt})^2 + \frac{kmc^2}{2} = mG\frac{4\pi}{3}r^3\frac{\rho}{r} \tag{2}$$

where $r = ar_0$ represents sphere radius, r_0 - appropriate length unit, while m can be considered as the mass of a classical probe system, a particle or spherical shell. Term $\frac{1}{2}(m\frac{dr}{dt})^2$ can be formally considered as the classical kinetic energy. Constant term $\frac{kmc^2}{2}$ can be formally considered as the total energy of a classical harmonic linear oscillator or rotator. Finally, term $mG(\frac{4\pi}{3})r^3\frac{\rho}{r}$ can be formally considered as the classical potential energy by gravitational interaction between probe system and universe with mass $(\frac{4\pi}{3})r^3\rho$ homogeneously distributed by density ρ within the sphere volume $(\frac{4\pi}{3})r^3$.

Friedmann equation with additional cosmological constant term

$$(\frac{1}{a}\frac{da}{dt})^2 + \frac{kc^2}{a^2} = G\frac{8\pi}{3}\rho + \frac{\Lambda}{3} \tag{3}$$

(where Λ represents the cosmological constant) can be, also, formally classically interpreted. Namely, it can be equivalently, simply transformed in the following equation

$$\frac{1}{2}(m\frac{dr}{dt})^2 + \frac{kmc^2}{2} = mG\frac{4\pi}{3}r^3\frac{\rho}{r} + m\frac{\Lambda}{24\pi}(4\pi r^2) \tag{4}$$

Additional term $m(\frac{\Lambda}{24\pi})(4\pi r^2)$ can be formally considered as the classical energy of the surface tension of a fluid captured in the sphere with radius r and surface area $(4\pi r^2)$ so that surface tension coefficient is $m(\frac{\Lambda}{24\pi})$. It is very important to be pointed out that here (as well as in the fluid mechanics), even if surface tension coefficient is constant, energy of the surface tension increases when sphere radius increases.

All this implies a possibility [2] that cosmological constant, not only formally classically, but even really corresponds to some surface phenomena. (Especially, it can be observed that all this corresponds, at least conceptually, to t'Hooft-Susskind holographic principle in quantum gravity [3].) One aspect of this possibility we shall roughly consider in this work. Namely, in this work a satisfactory, simplified theoretical prediction of the data corresponding to observationally (by fine tuning condition) estimated value of the cosmological constant [4], [5] will be given. It will be supposed that cosmological constant, like surface tension coefficient by a liquid drop, does not correspond to a volume (bulk) vacuum mass (energy) density distribution but that it corresponds to a surface vacuum mass (energy) density distribution. Precisely, it will be supposed that vacuum mass is distributed over a thin spherical shell with sphere radius proportional to scale factor and thickness equivalent to Planck length. Then form of given surface mass distribution and fine tuning condition imply observed value of the scale factor, i.e. growing of the scale factor (for ~ 61 magnitude order) from initial (corresponding to Planck length $\sim 10^{-35}m$) to recent (at the beginning of the cosmic acceleration, corresponding to $10Glyr \sim 10^{26}m$ length).

So, suppose, usually,

$$G\frac{8\pi}{3}\rho_\Lambda = \frac{\Lambda}{3} \tag{5}$$

where ρ_Λ represents the mass density corresponding to cosmological constant.

Further, suppose

$$\rho_\Lambda = \frac{M_\Lambda}{L_P 4\pi r^2} \tag{6}$$

which corresponds to vacuum mass M_Λ homogeneously distributed over thin spherical shell with sphere radius r and thickness equivalent to Planck length L_P. It implies

$$M_\Lambda = \rho_\Lambda(L_P 4\pi)r^2 \tag{7}$$

which means that vacuum mass grows quadratically when sphere radius grows.

Introduction of (6) and

$$\Lambda = \frac{c^2}{L_\Lambda^2} \tag{8}$$

(where L_Λ represents the length corresponding to cosmological constant) in (5), after simple transformations, yields

$$\frac{M_\Lambda}{M_P} = \frac{1}{2}\frac{r^2}{L_\Lambda^2} \tag{9}$$

It represents very interesting result admitable for comparison with observational data.

Initially, i.e. for $r = L_P$, and, according to fine-tuning condition [4], [5]

$$\frac{L_P^2}{L_\Lambda^2} \sim 10^{-123} \tag{10}$$

it follows

$$\frac{M_\Lambda}{M_P} = \frac{1}{2}\frac{L_P^2}{L_\Lambda^2} \tag{11}$$

or

$$M_\Lambda = 10^{-123} M_P \tag{12}$$

It means, of course, that cosmological constant does not any important influence in the early universe.

Now, we shall determine by (9) such r for which condition

$$M_\Lambda \sim M_P \tag{13}$$

is satisfied. Given condition, of course, simply means that vacuum energy becomes comparable with energy of the quantum fields. Introduction of (13) in (9) yields, after simple transformations,

$$r^2 \sim L_\Lambda^2 \tag{14}$$

Now, we shall express r in the following way

$$r = aL_P \tag{15}$$

which, introduced in (14), yields

$$a^2 \sim \frac{L_\Lambda^2}{L_P^2} \sim 10^{123} \tag{16}$$

and further

$$a \sim 10^{61} \tag{17}$$

It represents very important result. Namely, it satisfactorily corresponds to observational data [4]. [5] on the growing of the scale factor of the universe for ~ 61 magnitude order ,from initial (corresponding to Planck length $\sim 10^{-35}m$) to recent (at the beginning of the cosmic acceleration, corresponding to $\sim 10Glyr \sim 10^{26}m$ length). (It can be added that, as it is not hard to see, supposition on the volume, i.e. bulk distribution of the vacuum mass yields unsatisfactory prediction $a \sim 10^{41}$).

Finally, we can shortly repeated and pointed out the following. It can be pointed out that suggested model of the interpretation of cosmological constant as a surface phenomena (at least

conceptually analogous to t'Hooft-Susskind holographic principle) is very rough and simplified, formally like to a classical. Such model must be necessarily generalized, but it goes over basic intention of our work. In any case our model is able to correlate and reproduce observational astronomical data (fine-tuning and growing of the scale factor) in a satisfactory way. All this is very interesting and promising.

1 References

[1] B. W. Carroll, D. A. Ostlie, *An Introduction to Modern Galactic Astrophysics and Cosmology* (Addison-Wesley, Reading, MA, 2007)

[2] T. Padmanabhan, *Dark Energy: Mystery of the Millennium*, astro-ph/0603114 and references therein

[3] R. Bousso, *The holographic principle*, hep-th/0203101 and references therein

[4] D. N. Spergel et al., Astrophys. J. Supp. **146** (2003) 175 ; astro-ph/0302209.

[5] D. N. Spergel et al., Astrophys. J. Supp. **170** (2007) 337 ; astro-ph/0603449.